Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
и.о. директора физтех-школы физики и исследований им. Ландау
A.A. Voronov

Work program of the course (training module)

course:	Physics of Quantum Fluids/Физика квантовых жидкостей
major:	Photonics and Optical Informatics
specialization:	Photonics, Quantum Technologies & 2D Materials/Фотоника, квантовые технологии и двумерные материалы
	Landau Phystech-School of Physics & Research
	Chair of the Russian Quantum Centre
term:	1
qualification:	Master

Semester, form of interim assessment: 2 (spring) - Exam

Academic hours: 30 AH in total, including:
- lectures: 30 AH.
- seminars: 0 AH.
- laboratory practical: 0 AH.

Independent work: 30 AH.

Exam preparation: 30 AH.

In total: 90 AH, credits in total: 2

Author of the program: G.V. Shlyapnikov, doctor of physics and mathematical sciences

The program was discussed at the Chair of the Russian Quantum Centre 04.06.2020

Annotation

The course is dedicated to ultracold quantum gases/fluids and consists of two parts.

The first one discusses ultracold bosonic atoms and to the phenomenon of Bose-Einstein condensation. After the celebrated observation of Bose-Einstein condensation in trapped ultracold alkali atom gases in 1995, which got the Nobel Prize in 2001, the field of ultracold quantum gases is strongly expanding and it attracts scientists from atomic physics and quantum optics, condensed matter physics, nuclear physics, non-linear phenomena, and mathematical physics.

The second part of the lecturing course discusses ultracold degenerate gases of fermionic atoms and is focused on the role of interactions and on superfluidity. After creating quantum degenerate atomic Fermi gases, experiments have reached the so-called strongly interacting regime bringing analogies with neutron stars and high temperature superconductivity. Therefore, aside from an Introduction to the theory of degenerate fermions, the lecturing course includes several modern developments in this domain.

The purpose of the lecturing course as a whole is to give an Introduction to the theory of ultracold quantum gases/fluids and to provide students with necessary tools to work in this and related areas.

1. Study objective

Purpose of the course

Provide students with the basic knowledge of ultracold quantum systems, which are one of the main topics of research around the world and at RQC.

Tasks of the course

Provide understanding of the main results obtained in the physics of ultracold quantum systems.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-1 Use a systematic approach to critically analyze a problem and develop an action plan	UC-1.1 Systematically analyze the problem situation, identify its components and the relations between them
	UC-1.2 Search for solutions by using available sources
Gen.Pro.C-1 Gain fundamental scientific knowledge in the field of physical and mathematical sciences	Gen.Pro.C-1.1 Apply fundamental scientific knowledge in the field of physical and mathematical sciences
	Gen.Pro.C-1.3 Understands the interdisciplinary links in mathematics and physics and is able to apply them to problems in photonics and opto-informatics
Gen.Pro.C-2 Acquire an understanding of current scientific and technological challenges in professional settings, and scientifically formulate professional objectives	Gen.Pro.C-2.3 Understand professional terminology used in modern scientific and technical literature and present scientific results in oral and written form within professional communication
Gen.Pro.C-3 Select and/or develop approaches to professional problem-solving with consideration to the limitations and specifics of different solution methods	Gen.Pro.C-3.1 Analyze problems, plan research strategy to achieve solution(s), propose, and combine solution approaches
	Gen.Pro.C-3.2 Employ research methods to solve new problems, and apply knowledge from various fields of science (technology)
	Gen.Pro.C-3.3 Gain knowledge of analytical and computational methods of problem-solving, understand the limitations for applying the obtained solutions in practice
Gen.Pro.C-4 Successfully perform a task, analyze the results and present conclusions, apply knowledge and skills in the field of physical and mathematical sciences and ICTs	Gen.Pro.C-4.2 Apply knowledge in the field of physical and mathematical sciences to solve problems, make conclusions, and evaluate the obtained results

Pro.C-1 Assign, formalize, and solve tasks, develop and research mathematical models of the studied phenomena and processes, systematically analyze scientific problems and obtain new scientific results	Pro.C-1.2 Make hypotheses, build mathematical models of the studied phenomena and processes, evaluate the quality of the developed model

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

Primary concepts of the physics of ultracold quantum systems: Bose-Einstein condensation, Superfluidity in Bose and Fermi systems etc.

be able to:

solve a large variety of problems related to the physics of ultracold quantum systems.

master:

mathematical techniques that are necessarily for solving these problems.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Key quantities. Elastic and inelastic interaction between atoms.	4			4
2	Bose-Einstein condensation in an ideal gas.	2			2
3	Weakly interacting Bose gas. Gross-Pitaevskii equation.	2			2
4	Dynamics of Bose-Einstein condensates.	2			2
5	Elementary excitations of a Bose-condensed gas.	2			2
6	Bose-condensed gas at a finite temperature. Superfluidity in Bose systems.	2			2
7	Vortices in Bose-condensed gases.	2			2
8	Ideal Fermi gas. Thermodynamics and excitations.	2			2
9	Repulsively interacting Fermi gas. Landau’s Fermi liquid theory.	2			2
10	Attractively interacting Fermi gas. Superfluid pairing.	2			2
11	Superfluidity in Fermi gases.	2			2
12	Ginzburg-Landau approach. Vortices in Fermi gases.	2			2
13	Strongly interacting Fermi gases.	4			4
AH in total		30			30
Exam preparation		30 AH.			
Total complexity		90 AH., credits in total 2			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 2 (Spring)

1. Key quantities. Elastic and inelastic interaction between atoms.

Key quantities. Elastic interaction between atoms. Weakly interacting regime. Two-body scattering problem. Inelastic collisions

2. Bose-Einstein condensation in an ideal gas.

Second quantization. Thermodynamics of an ideal Fermi gas. Particle and hole excitations.

3. Weakly interacting Bose gas. Gross-Pitaevskii equation.

Gross-Pitaevskii equation for the condensate wavefunction. Density-phase representation. Collapsing condensates. Stable condensates. Healing length. Bose-Einstein condensation in an external harmonic potential.

4. Dynamics of Bose-Einstein condensates.

Exact scaling approach for a two-dimensional evolution of a trapped condensate. Scaling approach for evolving 3D trapped condensates. Fundamental frequencies of oscillating condensates.

5. Elementary excitations of a Bose-condensed gas.

Bogoliubov transformation. Excitation spectrum of a uniform condensate. Non-condensed fraction. One-body density matrix and long range order. Quantum fluctuations of the density and phase. Quantum fluctuations and ground state energy.

6. Bose-condensed gas at a finite temperature. Superfluidity in Bose systems.

Non-condensed fraction and the one-body density matrix at finite temperatures. Landau criterion of superfluidity. Superfluid and normal density. Beliaev damping of elementary excitations. Landau damping. Small parameter of the theory at finite temperatures.

7. Vortices in Bose-condensed gases.

Vortices in rotating and non-rotating superfluids. Circulation. Gross-Pitaevskii equation for the vortex state. Excitations of the vortex state. Fundamental modes. Kelvin modes and vortex contrast.

8. Ideal Fermi gas. Thermodynamics and excitations.

Second quantization. Thermodynamics of an ideal Fermi gas. Particle and hole excitations

9. Repulsively interacting Fermi gas. Landau's Fermi liquid theory.

Weakly interacting Fermi gas with repulsion between particles. Quasiparticles in Landau's Fermi liquid theory. Hydrodynamic regime. Collisionless regime. Zero sound.

10. Attractively interacting Fermi gas. Superfluid pairing.

Cooper problem. BCS approach. Gapped single-particle excitations. Order parameter and transition temperature.

11. Superfluidity in Fermi gases.

Landau criterion in Fermi gases. Superfluid current. Bogoliubov-Anderson sound. Superfluid and normal density. Thermodynamic quantities near Tc.

12. Ginzburg-Landau approach. Vortices in Fermi gases.

Landau-Ginzburg functional. Critical fluctuations. Vortex state. Vortices near Tc

13. Strongly interacting Fermi gases.

Anomalously large scattering length. Fano-Feshbach resonance. BCS-BEC crossover. Description of the strongly interacting regime. Unitarity limit.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

A classroom equipped with a board, multimedia-projector, and projection screen.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Теоретическая физика [Текст] : в 10 т. Т. 9, Ч. 2 : Статистическая физика. Теория конденсированного состояния : учеб. пособие для вузов / Л. Д. Ландау, Е. М. Лифщиц .— М. : Физматлит, 2000-2005 .— 496 с.

Additional literature

L. Pitaevskii and S. Stringari, Bose-Einstein Condensation. Oxford University Press, 2016.

7. List of web resources that are necessary for the course (training module) mastering

https://arxiv.org/

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

No.

9. Guidelines for students to master the course

Students should learn the basic concepts of quantum fluids, as well as how to apply their theoretical knowledge in practice.

For the successful assimilation of the course, in addition to attending classes, students are required to perform homework whose amount in hours should be not less than the number of hours specified in the curricula of the faculties. Studying at home includes:

- reading the recomended literature and making notes;
- processing and analysis of lecture materials (using notes, textbooks and scientific articles), answering questions, proving some statements;
- solving problems given on lectures and seminars for self-study;
- preparing for the tests and exams.

SUPPLEMENT

Assessment funds for course (training module)

major:	Photonics and Optical Informatics
specialization:	Photonics, Quantum Technologies & 2D Materials/Фотоника, квантовые технологии и двумерные материалы
	Landau Phystech-School of Physics & Research
	Chair of the Russian Quantum Centre
term:	1
qualification:	Master

Semester, form of interim assessment: 2 (spring) - Exam

Author:	G.V. Shlyapnikov, doctor of physics and mathematical sciences

1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
UC-1 Use a systematic approach to critically analyze a problem and develop an action plan	UC-1.1 Systematically analyze the problem situation, identify its components and the relations between them
	UC-1.2 Search for solutions by using available sources
Gen.Pro.C-1 Gain fundamental scientific knowledge in the field of physical and mathematical sciences	Gen.Pro.C-1.1 Apply fundamental scientific knowledge in the field of physical and mathematical sciences
	Gen.Pro.C-1.3 Understands the interdisciplinary links in mathematics and physics and is able to apply them to problems in photonics and opto-informatics
Gen.Pro.C-2 Acquire an understanding of current scientific and technological challenges in professional settings, and scientifically formulate professional objectives	Gen.Pro.C-2.3 Understand professional terminology used in modern scientific and technical literature and present scientific results in oral and written form within professional communication
Gen.Pro.C-3 Select and/or develop approaches to professional problem-solving with consideration to the limitations and specifics of different solution methods	Gen.Pro.C-3.1 Analyze problems, plan research strategy to achieve solution(s), propose, and combine solution approaches
	Gen.Pro.C-3.2 Employ research methods to solve new problems, and apply knowledge from various fields of science (technology)
	Gen.Pro.C-3.3 Gain knowledge of analytical and computational methods of problem-solving, understand the limitations for applying the obtained solutions in practice
Gen.Pro.C-4 Successfully perform a task, analyze the results and present conclusions, apply knowledge and skills in the field of physical and mathematical sciences and ICTs	Gen.Pro.C-4.2 Apply knowledge in the field of physical and mathematical sciences to solve problems, make conclusions, and evaluate the obtained results
Pro.C-1 Assign, formalize, and solve tasks, develop and research mathematical models of the studied phenomena and processes, systematically analyze scientific problems and obtain new scientific results	Pro.C-1.2 Make hypotheses, build mathematical models of the studied phenomena and processes, evaluate the quality of the developed model

2. Competency assessment indicators

As a result of studying the course the student should:

know:

Primary concepts of the physics of ultracold quantum systems: Bose-Einstein condensation, Superfluidity in Bose and Fermi systems etc.

be able to:

solve a large variety of problems related to the physics of ultracold quantum systems.

master:

mathematical techniques that are necessarily for solving these problems.

3. List of typical control tasks used to evaluate knowledge and skills

Examples of control questions:

1. Bose condensation in an ideal gas. The role of state density.
2. Ginzburg-Landau approximation. Description of vortices in Fermi gases.

Examples of control tasks:

1. Calculate the scattering amplitude in an interaction potential that has an absolutely absorbing wall at a distance of R0 and behaves as -1/R^4 at large distances.

4. Evaluation criteria

Checking questions:

1. Elastic interaction between atoms. Derivation of the expression for the constant g. Criterion of the weakly interacting regime.
2. Strongly interacting regime in Fermi gases. Description of the BCS-BEC crossover.
3. Vortex states in Bose-condensed gases. Solution of the stationary Gross-Pitaevskii equation for the vortex state (with angular momentum l = 1) in the uniform space, assuming a straight vortex line.
4. Superfluidity in Fermi gases. Landau criterion.
5. Bose-Einstein condensation in an ideal gas of bosons. Results for the uniform 3D, 2D and 1D cases and in the presence of an external harmonic potential. Role of the density of states.
6. Strongly interacting regime in Fermi gases. Description of the BCS-BEC crossover.
7. Elementary excitations of a Bose-condensed gas. Bogoliubov-de Gennes equations. Wave-function and spectrum of excitations of a uniform condensate.
8. Landau-Ginzburg functional. Vortices in a superfluid Fermi gas.

9 Hamiltonian for the weakly-interacting Bose gas in second quantization. Gross-Pitaevskii equation for the condensate wavefunction. Solution in the presence of an external (spherically symmetrical) harmonic potential. Thomas-Fermi regime.

10. BCS approach for attractively interacting fermions. Single-particle excitations, order parameter and transition temperature.

Examples of examination cards:

Card 1.

1. Elastic interaction between atoms. Derivation of the expression for the constant g. Criterion of the weakly interacting regime.
2. Strongly interacting regime in Fermi gases. Description of the BCS-BEC crossover.

Card 2.

1. Vortex states in Bose-condensed gases. Solution of the stationary Gross-Pitaevskii equation for the vortex state (with angular momentum l = 1) in the uniform space, assuming a straight vortex line.
2. Superfluidity in Fermi gases. Landau criterion.

Assessment “excellent (10)” is given to a student who has displayed comprehensive, systematic and deep knowledge of the educational program material, has independently performed all the tasks stipulated by the program, has deeply studied the basic and additional literature recommended by the program, has been actively working in the classroom, and understands the basic scientific concepts on studied discipline, who showed creativity and scientific approach in understanding and presenting educational program material, whose answer is characterized by using rich and adequate terms, and by the consistent and logical presentation of the material;

Assessment “excellent (9)” is given to a student who has displayed comprehensive, systematic knowledge of the educational program material, has independently performed all the tasks provided by the program, has deeply mastered the basic literature and is familiar with the additional literature recommended by the program, has been actively working in the classroom, has shown the systematic nature of knowledge on discipline sufficient for further study, as well as the ability to amplify it on one’s own, whose answer is distinguished by the accuracy of the terms used, and the presentation of the material in it is consistent and logical;

Assessment “excellent (8)” is given to a student who has displayed complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently performed all the tasks stipulated by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (7)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in the answer, has independently performed all the tasks provided by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (6)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently carried out the main tasks stipulated by the program, studied the basic literature recommended by the program, showed systematic character of his knowledge of the discipline, which is sufficient for further study;

Assessment “good (5)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, mastered the basic literature recommended by the program, made some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors by himself;

Assessment “satisfactory (4)” is given to a student who has discovered knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, learned the main literature but allowed some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors under the guidance of a teacher;

Assessment “satisfactory (3)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, not showed activity in the classroom, independently fulfilled the main tasks envisaged by the program, but allowed errors in their implementation and in the answer during the test, but possessing necessary knowledge for elimination under the guidance of the teacher of the most essential errors;

Assessment “unsatisfactory (2)” is given to a student who showed gaps in knowledge or lack of knowledge on a significant part of the basic educational program material, who has not performed independently the main tasks demanded by the program, made fundamental errors in the fulfillment of the tasks stipulated by the program, who is not able to continue his studies or start professional activities without additional training in the discipline in question;

Assessment “unsatisfactory (1)” is given to a student when there is no answer (refusal to answer), or when the submitted answer does not correspond at all to the essence of the questions contained in the task.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

The course is graded at an oral exam. The exam starts with a random task assigned to each student and time given for completion of the task. No aids are allowed. The student then proceeds to a chat with the examiner, at which he/she presents his/her solution to the assigned task. The examiner then asks the student several questions that evenly cover the course content. A final grade is assigned based on the quality of answers and demonstrated level of understanding.