Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Head of Landau Phystech-School of Physics & Research
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Work program of the course (training module)

course:	Fundamentals of Optical Radiation Propagation and Scattering/Основы распространения и рассеяния оптического излучения
major:	Photonics and Optical Informatics
specialization:	Photonics, Quantum Technologies & 2D Materials/Фотоника, квантовые технологии и двумерные материалы
	Landau Phystech-School of Physics & Research
	Chair of Physics and Technology of nanostructures
term:	1
qualification:	Master

Semesters, forms of interim assessment:

1 (fall) - Grading test
2 (spring) - Exam

Academic hours: 60 AH in total, including:
lectures: 60 AH.
seminars: 0 AH.
laboratory practical: 0 AH.

Independent work: 45 AH.

Exam preparation: 30 AH.

In total: 135 AH, credits in total: 3

Author of the program: D.G. Baranov, candidate of physics and mathematical sciences

The program was discussed at the Chair of Physics and Technology of nanostructures 01.02.2021

Annotation

The goal of the discipline is to provide students with a general understanding of the fundamental aspects of the propagation, radiation, and scattering of light in particular, and electromagnetic radiation in general. The students will be taught the general approach to describing light scattering by objects and nanostructures, and the universal patterns observed in the interaction of optical radiation with resonant nanostructures will be shown.

1. Study objective

Purpose of the course

To provide students with a general understanding of the fundamental aspects of the propagation, radiation, and scattering of light in particular, and electromagnetic radiation in general. Teach students the universal methods of describing light scattering by objects and nanostructures, and demonstrate the universal patterns observed in the interaction of optical radiation with resonant nanostructures.

Tasks of the course

- Mastering the basics of electromagnetism
- Teach methods for describing the propagation and emission of electromagnetic radiation in homogeneous space and waveguides
- Teach students the universal methods for describing the scattering of optical radiation by resonant nanostructures
- Gaining knowledge about the geometric and polarization characteristics of light and their transformation upon scattering by objects
- Teaching students the skill of employing the learned methods for solving practical problems

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-1 Use a systematic approach to critically analyze a problem and develop an action plan	UC-1.1 Systematically analyze the problem situation, identify its components and the relations between them
	UC-1.2 Search for solutions by using available sources
Gen.Pro.C-1 Gain fundamental scientific knowledge in the field of physical and mathematical sciences	Gen.Pro.C-1.1 Apply fundamental scientific knowledge in the field of physical and mathematical sciences
	Gen.Pro.C-1.2 Able to summarise and critically evaluate experiences and research results in the field of photonics and opto-informatics
	Gen.Pro.C-1.3 Understands the interdisciplinary links in mathematics and physics and is able to apply them to problems in photonics and opto-informatics
Gen.Pro.C-3 Select and/or develop approaches to professional problem-solving with consideration to the limitations and specifics of different solution methods	Gen.Pro.C-3.1 Analyze problems, plan research strategy to achieve solution(s), propose, and combine solution approaches
	Gen.Pro.C-3.3 Gain knowledge of analytical and computational methods of problem-solving, understand the limitations for applying the obtained solutions in practice
Gen.Pro.C-4 Successfully perform a task, analyze the results and present conclusions, apply knowledge and skills in the field of physical and mathematical sciences and ICTs	Gen.Pro.C-4.1 Apply ICT knowledge and skills to find and study scientific literature and use software products
	Gen.Pro.C-4.2 Apply knowledge in the field of physical and mathematical sciences to solve problems, make conclusions, and evaluate the obtained results
	Gen.Pro.C-4.3 Justify the chosen method of scientific research
Pro.C-1 Assign, formalize, and solve tasks	Pro.C-1.1 Locate, analyze, and summarize information on current research findings within a selected subject field

Pro.C-1 Assign, formalize, and solve tasks, develop and research mathematical models of the studied phenomena and processes, systematically analyze scientific problems and obtain new scientific results	Pro.C-1.2 Make hypotheses, build mathematical models of the studied phenomena and processes, evaluate the quality of the developed model
	Pro.C-1.3 Able to apply theoretical and/or experimental research methods in photonics and opto-informatics to a specific scientific problem and interpret the results obtained
Pro.C-2 Organize and conduct scientific research and testing independently or as a member (leader) of a small research team	Pro.C-2.1 Able to plan and carry out research in photonics and opto-informatics independently or as part of a research team
Pro.C-3 Professionally use research and testing equipment (devices and installations, specialized software) in a selected subject field	Pro.C-3.1 Understand the operating principles of the equipment and specialized software
	Pro.C-3.3 Evaluate the accuracy of the experimental (numerical) results

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- Fundamentals of the mathematical apparatus for describing the electromagnetic field (Maxwell's equations, wave equation, Green's tensor, Poynting's theorem, scattering matrix)
- Basic approaches to describing the propagation and scattering of optical radiation
- Various classes of localized solutions (waveguide modes, leaky modes, resonances, bound states within the continuum) and classes of resonant optical effects observed in resonant nanostructures

be able to:

- Find the dispersion laws of optical modes of homogeneous media and waveguide structures
- Calculate the fields of the simplest radiating systems in a homogeneous medium
- Calculate eigenmodes and eigenfrequencies of basic nanostructures (layers, cylinders, spheres)
- Simulate light scattering by an arbitrary resonant nanostructure at a basic level

master:

- General methods for solving problems of propagation of radiation of electromagnetic waves (search for the spectrum of waveguide modes, calculation of radiation);
- Methods of searching for optical eigenmodes and natural frequencies of resonant nanostructures
- Analytical methods for describing the scattering of optical radiation by generalized resonant structures

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Maxwell's equations, material relations, wave equation	2			1
2	Radiation problem, Green's tensor, eigenmodes	2			1
3	Symmetries and conservation laws in optics	2			1
4	Plane waves in homogeneous media; isofrequencies	2			1
5	Scattering of waves at the interface; transfer matrix method	2			1
6	Standing waves, cylindrical beams, spherical harmonics	2			1

7	Waveguiding structures; waveguide mode classes	2			1
8	Waveguide modes of planar and cylindrical systems	2			1
9	Surface waves; waveguiding in a thin layer	2			1
10	Modes of periodic structures; photonic crystals	2			1
11	Radiation problem; Green's tensor of free space; dipole radiation	2			1
12	Multipole decomposition; radiation near the surface	2			1
13	Radiation intensity, density of states	2			1
14	Scattering problem, Lippmann-Schwinger equation; scattering matrix	2			1
15	Eigenmodes, resonances; zeros and poles of the scattering matrix	2			1
16	Coupled mode theory	4			3
17	Exactly solvable scattering problems	4			3
18	Non-Hermitian optics: absorbers and lasers	2			3
19	Bound states within the continuum	2			2
20	Exceptional points	2			2
21	Scattering by a compact object; scattering cross sections; sphere scattering	4			3
22	Cloaking and super-scattering	2			3
23	Coupled dipole method; diffraction by arrays	2			2
24	Light polarization; Jones matrices	2			2
25	Spin and orbital moment; chirality of light	2			2
26	Polarization conversion; classification of polarization effects	2			2
27	Classification of magneto-electric media; simple chiral media	2			3
AH in total		60			45
Exam preparation		30 AH.			
Total complexity		135 AH., credits in total 3			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 1 (Fall)

1. Maxwell's equations, material relations, wave equation

Basic mathematical relations of electromagnetism: Maxwell's equations, wave equation, Helmholtz equation. Harmonic form of equations. Equivalence principle. Material relations, Lorentz and Debye models. Causality.

2. Radiation problem, Green's tensor, eigenmodes

Maxwell's equations with a source, formulation of the radiation problem. Green's function concept. Eigenmodes, expansion of the Green's function of closed structures in terms of eigenmodes.

3. Symmetries and conservation laws in optics

Poynting's theorem, Poynting's vector, energy conservation law. T-invariance and reciprocity of electromagnetic systems. Scaling invariance principle.

4. Plane waves in homogeneous media; isofrequencies

Solutions of Maxwell's equations in homogeneous isotropic space. Plane waves; evanescent waves. Waves in media with a negative refractive index. Dispersion of waves in anisotropic materials, Fresnel equation.

5. Scattering of waves at the interface; transfer matrix method

Transmission and reflection of a plane wave at the interface between two media, Fresnel's formula. Transfer matrix method for isotropic media.

6. Standing waves, cylindrical beams, spherical harmonics

Superposition of plane waves, standing waves. Non-diffracting beams, Bessel beams. Scalar and vector spherical harmonics.

7. Waveguiding structures; waveguide mode classes

The problem of waveguide modes. Classification of waveguide modes: localized, leaky, and anti-waveguide modes.

8. Waveguide modes of planar and cylindrical systems

Waveguide modes of a flat and cylindrical metal waveguide. Dielectric layer and cylinder modes.

9. Surface waves; waveguiding in a thin layer

Waveguide solutions at the interface between two media. Wave conduction by a thin conductive layer. Dyakonov waves.

10. Modes of periodic structures; photonic crystals

Waveguide modes of periodic structures, Bloch's theorem, photonic crystals, band gap.

11. Radiation problem; Green's tensor of free space; dipole radiation

Statement of the radiation problem. Finding the Green's tensor for a homogeneous isotropic space. Radiation of an electric and magnetic dipole.

12. Multipole decomposition; radiation near the surface

Decomposition of the field of the radiating system into spherical harmonics. Multipole decomposition of current. Radiation of a dipole near the interface between two media.

13. Radiation intensity, density of states

The power of the dipole radiation, the relationship with the density of states, the Purcell factor.

14. Scattering problem, Lippmann-Schwinger equation; scattering matrix

Statement of the scattering problem, Lippmann-Schwinger integral equation; scattering channels, scattering matrix.

15. Eigenmodes, resonances; zeros and poles of the scattering matrix

The concept of eigenmodes and resonances. Complex frequency plane, natural frequencies; zeros and poles of the scattering matrix.

Semester: 2 (Spring)

16. Coupled mode theory

Phenomenological theory of coupled modes for describing the response of resonant systems. The case of several modes and several scattering channels.

17. Exactly solvable scattering problems

Scattering matrices of the interface, layer, eigenvalues and eigenmodes Connection of complex eigenmodes with the waveguide problem.

18. Non-Hermitian optics: absorbers and lasers

Physics of systems with attenuation and amplification; ideal absorbers, coherent absorbers; linear theory of lasers.

19. Bound states within the continuum

Physics of bound states in the continuum, methods of their occurrence, modeling within the framework of the theory of coupled modes.

20. Exceptional points

Singular points of Hamiltonians, examples of singular points in non-Hermitian systems. PT symmetry, laser absorber.

21. Scattering by a compact object; scattering cross sections; sphere scattering

Description of field scattering by a compact object. Decomposition of a plane wave in spherical harmonics. Scattering cross sections, scattering amplitude. Optical theorem. The problem of scattering by a sphere, resonances of spheres.

22. Cloaking and super-scattering

Suppression of scattering by a compact object; anapol. Super-scattering by nanoparticles.

23. Coupled dipole method; diffraction by arrays

Coupled dipole method. Scattering by two bound atoms. Light scattering by a periodic array, diffraction orders, diffraction singularities.

24. Light polarization; Jones matrices

Polarization of the electromagnetic field, polarization ellipse, Stokes parameters, Poincaré sphere. Jones matrices.

25. Spin and orbital moment; chirality of light

Angular moment of light, division into spin and orbital angular momentum. Spin-orbital coupling. Density of chirality, chirality operator. Dual structures. Relationship between chirality and spin.

26. Polarization conversion; classification of polarization effects

Polarization effects in the interaction of light with planar periodic structures. Symmetry classification of periodic structures.

27. Classification of magneto-electric media; simple chiral media

Classification of magneto-electric media. The case of a bi-isotropic chiral medium, rotation of polarization and circular dichroism.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

A set of electronic presentations/slides; an audience equipped with presentation equipment (projector, screen, computer / laptop); if necessary, special technical means for students with disabilities and persons with disabilities.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Теоретическая физика [Текст] : в 10 т. Т. 2 : Теория поля : учеб. пособие для вузов / Л. Д. Ландау, Е. М. Лифшиц ; под. ред. Л. П. Питаевского .— 8-е изд., стереотип. — М. : Физматлит, 2001, 2003, 2006, 2012, 2014 .— 536 с.
2. Основы нанооптики [Текст] / Л. Новотный, Б. Хехт ; пер. с англ. А. А. Коновко, О. А. Шутовой ; под ред. В. В. Самарцева - М.Физматлит,2009, 2011

Literature fund of the basic departament:

1. Jackson, John David. "Classical electrodynamics." (1999): 841-842.

Additional literature

Literature fund of the basic departament:

1. Molding the flow of light / Joannopoulos, J. D., Johnson, S. G., Winn, J. N., & Meade, R. D. Princeton Univ Press, Princeton, NJ (2008).

7. List of web resources that are necessary for the course (training module) mastering

http://www.nanophotonics.es/widgets2.

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

When preparing and conducting lectures, the Internet is used.
In addition, Libre Office is used, as well as the Ink Scape graphics package.

9. Guidelines for students to master the course

A student studying the discipline must, on the one hand, master the general conceptual apparatus, and on the other hand, must learn to apply theoretical knowledge in practice.
As a result of studying the discipline, the student must know the basic definitions and concepts, be able to apply the knowledge gained to solve various problems.
Successful completion of the course requires:
- attendance of all classes provided for by the curriculum for the discipline;
- keeping a synopsis of classes;

- student's intense independent work.

Independent work includes:

- reading recommended literature;
- study of educational material, preparation of answers to questions intended for independent study;
- solving problems offered to students in the classroom;
- preparation for the performance of tasks of the intermediate certification.

An indicator of mastery of the material is the ability to answer questions on the topics of the discipline without a synopsis.

It is important to achieve an understanding of the material being studied, not its mechanical memorization. If a student finds it difficult to study certain topics, questions, he/she should seek advice from a teacher.

Intermediate control of students' knowledge is possible in the form of solving problems in accordance with the topic of classes.

SUPPLEMENT

Assessment funds for course (training module)

major:	Photonics and Optical Informatics
specialization:	Photonics, Quantum Technologies & 2D Materials/Фотоника, квантовые технологии и двумерные материалы
	Landau Phystech-School of Physics & Research
	Chair of Physics and Technology of nanostructures
term:	1
qualification:	Master

Semesters, forms of interim assessment:

1 (fall) - Grading test

2 (spring) - Exam

Author: D.G. Baranov, candidate of physics and mathematical sciences

1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
UC-1 Use a systematic approach to critically analyze a problem and develop an action plan	UC-1.1 Systematically analyze the problem situation, identify its components and the relations between them
	UC-1.2 Search for solutions by using available sources
Gen.Pro.C-1 Gain fundamental scientific knowledge in the field of physical and mathematical sciences	Gen.Pro.C-1.1 Apply fundamental scientific knowledge in the field of physical and mathematical sciences
	Gen.Pro.C-1.2 Able to summarise and critically evaluate experiences and research results in the field of photonics and opto-informatics
	Gen.Pro.C-1.3 Understands the interdisciplinary links in mathematics and physics and is able to apply them to problems in photonics and opto-informatics
Gen.Pro.C-3 Select and/or develop approaches to professional problem-solving with consideration to the limitations and specifics of different solution methods	Gen.Pro.C-3.1 Analyze problems, plan research strategy to achieve solution(s), propose, and combine solution approaches
	Gen.Pro.C-3.3 Gain knowledge of analytical and computational methods of problem-solving, understand the limitations for applying the obtained solutions in practice
Gen.Pro.C-4 Successfully perform a task, analyze the results and present conclusions, apply knowledge and skills in the field of physical and mathematical sciences and ICTs	Gen.Pro.C-4.1 Apply ICT knowledge and skills to find and study scientific literature and use software products
	Gen.Pro.C-4.2 Apply knowledge in the field of physical and mathematical sciences to solve problems, make conclusions, and evaluate the obtained results
	Gen.Pro.C-4.3 Justify the chosen method of scientific research
Pro.C-1 Assign, formalize, and solve tasks, develop and research mathematical models of the studied phenomena and processes, systematically analyze scientific problems and obtain new scientific results	Pro.C-1.1 Locate, analyze, and summarize information on current research findings within a selected subject field
	Pro.C-1.2 Make hypotheses, build mathematical models of the studied phenomena and processes, evaluate the quality of the developed model
	Pro.C-1.3 Able to apply theoretical and/or experimental research methods in photonics and opto-informatics to a specific scientific problem and interpret the results obtained
Pro.C-2 Organize and conduct scientific research and testing independently or as a member (leader) of a small research team	Pro.C-2.1 Able to plan and carry out research in photonics and opto-informatics independently or as part of a research team
Pro.C-3 Professionally use research and testing equipment (devices and installations, specialized software) in a selected subject field	Pro.C-3.1 Understand the operating principles of the equipment and specialized software
	Pro.C-3.3 Evaluate the accuracy of the experimental (numerical) results

2. Competency assessment indicators

As a result of studying the course the student should:

know:

- Fundamentals of the mathematical apparatus for describing the electromagnetic field (Maxwell's equations, wave equation, Green's tensor, Poynting's theorem, scattering matrix)
- Basic approaches to describing the propagation and scattering of optical radiation
- Various classes of localized solutions (waveguide modes, leaky modes, resonances, bound states within the continuum) and classes of resonant optical effects observed in resonant nanostructures

be able to:

- Find the dispersion laws of optical modes of homogeneous media and waveguide structures
- Calculate the fields of the simplest radiating systems in a homogeneous medium
- Calculate eigenmodes and eigenfrequencies of basic nanostructures (layers, cylinders, spheres)
- Simulate light scattering by an arbitrary resonant nanostructure at a basic level

master:

- General methods for solving problems of propagation of radiation of electromagnetic waves (search for the spectrum of waveguide modes, calculation of radiation);
- Methods of searching for optical eigenmodes and natural frequencies of resonant nanostructures
- Analytical methods for describing the scattering of optical radiation by generalized resonant structures

3. List of typical control tasks used to evaluate knowledge and skills

Not provided.

4. Evaluation criteria

Assessment “excellent (10)” is given to a student who has displayed comprehensive, systematic and deep knowledge of the educational program material, has independently performed all the tasks stipulated by the program, has deeply studied the basic and additional literature recommended by the program, has been actively working in the classroom, and understands the basic scientific concepts on studied discipline, who showed creativity and scientific approach in understanding and presenting educational program material, whose answer is characterized by using rich and adequate terms, and by the consistent and logical presentation of the material;

Assessment “excellent (9)” is given to a student who has displayed comprehensive, systematic knowledge of the educational program material, has independently performed all the tasks provided by the program, has deeply mastered the basic literature and is familiar with the additional literature recommended by the program, has been actively working in the classroom, has shown the systematic nature of knowledge on discipline sufficient for further study, as well as the ability to amplify it on one’s own, whose answer is distinguished by the accuracy of the terms used, and the presentation of the material in it is consistent and logical;

Assessment “excellent (8)” is given to a student who has displayed complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently performed all the tasks stipulated by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (7)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in the answer, has independently performed all the tasks provided by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (6)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently carried out the main tasks stipulated by the program, studied the basic literature recommended by the program, showed systematic character of his knowledge of the discipline, which is sufficient for further study;

Assessment “good (5)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, mastered the basic literature recommended by the program, made some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors by himself;

Assessment “satisfactory (4)” is given to a student who has discovered knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, learned the main literature but allowed some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors under the guidance of a teacher;

Assessment “satisfactory (3)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, not showed activity in the classroom, independently fulfilled the main tasks envisaged by the program, but allowed errors in their implementation and in the answer during the test, but possessing necessary knowledge for elimination under the guidance of the teacher of the most essential errors;

Assessment “unsatisfactory (2)” is given to a student who showed gaps in knowledge or lack of knowledge on a significant part of the basic educational program material, who has not performed independently the main tasks demanded by the program, made fundamental errors in the fulfillment of the tasks stipulated by the program, who is not able to continue his studies or start professional activities without additional training in the discipline in question;

Assessment “unsatisfactory (1)” is given to a student when there is no answer (refusal to answer), or when the submitted answer does not correspond at all to the essence of the questions contained in the task.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

The course is graded at a credit (9th term) and an exam (10th term). The questioning starts with a random task assigned to each student and time given for completion of the task. No aids are allowed. The student then proceeds to a chat with the examiner, at which he/she presents his/her solution to the assigned task. The examiner then asks the student several questions that evenly cover the course content. A final grade is assigned based on the quality of answers and demonstrated level of understanding.

List of checking questions (9th term):

1. Material relations in electrodynamics, models of material relations.
2. Poynting's theorem, energy flux, dissipation power.
3. Time reversal and scaling invariance in electrodynamics.
4. Spectrum of plane waves in an isotropic medium, evanescent waves. Isofrequency.
5. Scattering of a plane wave by the interface between two media, Fresnel's formula
6. Non-diffracting beams; Bessel beams.
7. Wave-guiding structures; classification of waveguide modes; light cone.
8. Spectrum of waveguide modes of the dielectric layer
9. Spectrum of surface waves at the metal-dielectric interface.
10. Photonic crystals. Band structure of a one-dimensional photonic crystal, band gap.
11. Intensity of dipole radiation, relationship with the density of states. The Purcell factor.
12. Statement of the scattering problem, Lippmann-Schwinger integral equation, scattering matrix.
13. Eigenmodes and natural frequencies. Properties of the scattering matrix at eigenfrequencies.

Examples of problems (9th term):

1. Calculate the Poynting vector of a y-polarized TE evanescent wave propagating along x $\mathbf{E}(\mathbf{r}) = \mathcal{E}\hat{\mathbf{y}}e^{ik_x x}e^{-\kappa_z z}$. Show that the z-component of the energy flux in such a wave is equal to 0.
2. Consider a linear combination of two evanescent waves propagating along x with the same wavenumber: $\mathbf{E}_1(\mathbf{r}) = \mathcal{E}\hat{\mathbf{y}}e^{ik_x x}e^{-\kappa_z z}$, $\mathbf{E}_2(\mathbf{r}) = \mathcal{E}e^{i\phi}\hat{\mathbf{y}}e^{ik_x x}e^{+\kappa_z z}$, $\mathbf{E}(\mathbf{r}) = \mathbf{E}_1(\mathbf{r}) + \mathbf{E}_2(\mathbf{r})$. Calculate the z-component of the Poynting vector as a function of the relative phase ϕ. What does this result say about the transfer of energy by evanescent waves?
3. Consider the propagation of plane waves in an isotropic medium described by the Lorentzian dielectric constant $\varepsilon(\omega) = \varepsilon_\infty + \frac{\omega_P^2}{\omega_0^2-\omega^2-i\gamma\omega}$. Plot the spectrum of solutions of plane waves k = k (ω). What happens to the real and imaginary parts of the wave vector near the resonance of the medium?
4. Find the solution to the same problem numerically, but in the complex frequency plane, assuming real k. Analyze the resulting spectrum of complex ω-solutions ω (k). What happens to the spectrum near the resonance of the medium?
5. Consider the reflection of a plane linearly polarized wave incident from air onto a metal without loss described by the Drude permittivity $\varepsilon = 1 - \frac{\omega_{pl}^2}{\omega^2}$, with an electric field polarized at an angle ϕ relative to the plane of incidence. Find the polarization state of the reflected wave. Tip: decompose the incident wave into s- and p-polarized components and calculate each reflection of each component separately.
6. Using the Bloch expansion of the stationary solution of the wave equation in a two-dimensional periodic system with periods $\mathbf{\Lambda} = (L_x, L_y)$,, show that for frequencies above $\overline{\omega} = \frac{2\pi c}{\max\{L_x, L_y\}}$ the system does not support true waveguide localized solutions.
7. Using the expression for the quasi-static potential of a point electric dipole $\phi = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{pr}}{|\mathbf{r}|^3}$, calculate the field created by such a dipole and the radiation intensity of the quasi-static dipole using the expression for the density of states $\mathrm{Im}[\mathbf{p}^* \cdot \mathbf{E}_{rad}(\mathbf{0})]$.

List of checking questions (10th term):

1. Construction of the theory of coupled modes for a single-mode resonator with one port.
2. Resonances of the dielectric layer.
3. Absorbers and lasers. Description of the absorber using coupled mode theory, critical coupling mode.
4. Bound states within the continuum.
5. Degenerate points in the spectrum of non-Hermitian systems. An example of structures with degenerate points.
6. Scattering of a plane wave by a compact object. Scattering cross sections.
7. Cloaking and super-scattering by a compact object.
8. The method of coupled dipoles.
9. Chirality of light, connection with spin.
10. Conversion of polarization in the interaction of light with planar periodic structures. Specific examples of unit cells falling into different symmetry classes.

Examples of problems (9th term):

1. Construct a formal scattering matrix for a vacuum layer of thickness L in the basis of plane waves propagating along the normal to this "vacuum layer". Find the eigenvalues of the given scattering matrix.
2. Consider a generalized single-mode resonant optical system associated with a single scattering channel and plot the phase (argument) of the complex scattering coefficient in the complex frequency plane. In which areas of the graph are phase gradients visible? What points have a topological charge?
3. Consider a single-mode resonant structure with γ_ (n-rad) dissipation symmetrically coupled to two scattering channels (simulating, for example, a dielectric layer in a symmetric environment). Using coupled mode theory for this structure, calculate the absorption coefficient when the system is excited from one channel. Find the condition for maximum absorption. What is the significance of this maximum absorption rate?
4. Now resolve the excitation of the system from the previous exercise through both diffusion channels. What is the maximum possible absorption level? What vector of the incident field s ^ + realizes this maximum absorption?

5. Consider a compact resonant two-mode lossless system with eigenmodes $\omega_1 - i\gamma_1$ and $\omega_2 - i\gamma_2$, where γ_i are the rates of purely radiative decay. Assume that both modes are predominantly associated with the same scattering channel (some multipole harmonic). Using coupled mode theory for a compact scatterer, calculate the scattering cross section of an object and demonstrate the invisibility effect. Does the effect persist in the presence of dissipation in the object?

Example of examination question papers:
Question paper 1.
1. Construction of the theory of coupled modes for a single-mode resonator with one port.
2. Consider a single-mode resonant structure with γ_ (n-rad) dissipation symmetrically coupled to two scattering channels (simulating, for example, a dielectric layer in a symmetric environment). Using coupled mode theory for this structure, calculate the absorption coefficient when the system is excited from one channel. Find the condition for maximum absorption. What is the significance of this maximum absorption rate?

Question paper 2.
1. Resonances of the dielectric layer.
2. Construct a formal scattering matrix for a vacuum layer of thickness L in the basis of plane waves propagating along the normal to this "vacuum layer". Find the eigenvalues of the given scattering matrix.

10.Фонд оценочных средств

Перечень контрольных вопросов в 9-ом семестре:

1. Материальные соотношения в электродинамике, модели материальных соотношений.
2. Теорема Пойнтинга, поток энергии, мощность диссипации.
3. Обращение времени и скейлинг-инвариантность в электродинамике.
4. Спектр плоских волн в изотропной среде, эванесцентные волны. Изочастота.
5. Рассеяние плоской волны границей раздела двух сред, формулы Френеля
6. Недифрагирующие пучки; Бесселевы пучки.
7. Волноведущие структуры; классификация волноводных мод; световой конус.
8. Спектр волноводных мод диэлектрического слоя
9. Спектр поверхностных волн на границе раздела металл-диэлектрик.
10. Фотонные кристаллы. Зонная структура одномерного фотонного кристалла, запрещенная зона.
11. Интенсивность дипольного излучения, связь с плотностью состояний. Фактор Парселла.
12. Постановка задачи рассеяния, интегральное уравнение Липпмана-Швингера, матрица рассеяния.
13. Собственные моды и собственные частоты. Свойства матрицы рассеяния на собственных частотах.

Примеры контрольных заданий в 9-ом семестре:

1. Вычислите вектор Пойнтинга y-поляризованной TE эванесцентной волны распространяющейся вдоль x $\mathbf{E}(\mathbf{r}) = \mathcal{E}\hat{\mathbf{y}}e^{ik_x x}e^{-\kappa_z z}$. Покажите, что z-компонента потока энергии в такой волне равна 0.
2. Рассмотрите линейную комбинацию двух эванесцентных волн распространяющихся вдоль x с одинаковым волновым числом: $\mathbf{E}_1(\mathbf{r}) = \mathcal{E}\hat{\mathbf{y}}e^{ik_x x}e^{-\kappa_z z}$, $\mathbf{E}_2(\mathbf{r}) = \mathcal{E}e^{i\phi}\hat{\mathbf{y}}e^{ik_x x}e^{+\kappa_z z}$, $\mathbf{E}(\mathbf{r}) = \mathbf{E}_1(\mathbf{r}) + \mathbf{E}_2(\mathbf{r})$. Вычислите z-компоненту вектора Пойнтинга как функцию относительной фазы ϕ. Что данный результат говорит о переносе энергии эванесцентными волнами?
3. Рассмотрите распространение плоских волн в изотропной среде, описываемой Лоренцевской диэлектрической проницаемостью $\varepsilon(\omega) = \varepsilon_\infty + \frac{\omega_p^2}{\omega_0^2-\omega^2-i\gamma\omega}$. Постройте спектр решений плоских волн k = k (ω). Что происходит с действительной и мнимой частью волнового вектора вблизи резонанса среды?
4. Численно найдите решение той же проблемы, но в комплексной частотной плоскости, предполагая действительное k. Проанализируйте полученный спектр комплексных ω-решений ω(k). Что происходит со спектром вблизи резонанса среды?
5. Рассмотрите отражение плоской линейно поляризованной волны, падающей из воздуха на металл без потерь описываемый проницаемостью Друде ($\varepsilon = 1 - \frac{\omega_{pl}^2}{\omega^2}$), с электрическим полем, поляризованным под углом ϕ относительно плоскости падения. Найдите состояние поляризации отраженной волны. Совет: разложите падающую волну на s- и p-поляризованные компоненты и рассчитайте каждое отражение каждой компоненты отдельно.
6. Используя разложение Блоха стационарного решения волнового уравнения в двумерной периодической системе с периодами $\mathbf{\Lambda} = (L_x, L_y)$, покажите, что для частот выше $\overline{\omega} = \frac{2\pi c}{\max\{L_x, L_y\}}$ система не поддерживает настоящих волноводных локализованных решений.
7. Используя выражение для квази-статического потенциала точечного электрического диполя $\phi = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{pr}}{|\mathbf{r}|^3}$, вычислите поле, создаваемое таким диполем, и интенсивность излучения квази-статического диполя, используя выражение для плотности состояний $\mathrm{Im}[\mathbf{p}^* \cdot \mathbf{E}_{rad}(\mathbf{0})]$.

Перечень контрольных вопросов в 10-ом семестре:

1. Построение теории связанных мод для одномодового резонатора с одним портом.
2. Резонансы диэлектрического слоя.
3. Поглотители и лазеры. Описание поглотителя с помощью теории связанных мод, режим критической связи.
4. Связанные состояния внутри континуума.
5. Вырожденные точки в спектре неэрмитовых систем. Пример структур с вырожденными точками.
6. Рассеяние плоской волны компактным объектом. Сечения рассеяния.
7. Клоакинг и суперрассеяние компактным объектом.
8. Метод связанных диполей.
9. Киральность света, связь со спином.
10. Конверсия поляризации при взаимодействии света с планарными периодическими структурами. Конкретные примеры элементарных ячеек, попадающих в разные классы симметрии.

Примеры контрольных заданий в 10-ом семестре:

1. Постройте формальную матрицу рассеяния для слоя вакуума толщиной L в базисе плоских волн распространяющихся по нормали к этому «слою вакуума». Найдите собственные числа данной матрицы рассеяния.
2. Рассмотрите обобщенную одномодовую резонансную оптическую систему, связанную с единственным каналом рассеяния, и постройте фазу (аргумент) комплексного коэффициента рассеяния в комплексной плоскости частот. В каких областях графика видны фазовые градиенты? Какие точки обладают топологическим зарядом?

3. Рассмотрите одномодовую резонансную структуру с диссипацией $\gamma\_$ (n-rad), симметрично связанной с двумя каналами рассеяния (моделирующую, например, диэлектрический слой в симметричном окружении). Используя теорию связанных мод для этой структуры, рассчитайте коэффициент поглощения при возбуждении системы из одного канала. Найдите условие максимального поглощения. Каково значение этой максимальной скорости поглощения?
4. Теперь разрешите возбуждение системы из предыдущего упражнения посредством обоих каналов рассеяния. Каков при этом максимально возможный уровень поглощения? Какой вектор падающего поля s^{inc} реализует это максимальное поглощение?
5. Рассмотрите компактную резонансную двухмодовую систему без потерь с собственными модами $\omega_1 - i\gamma_1$ и $\omega_2 - i\gamma_2$, где γ_i - скорости чисто радиационного распада. Предположите, что обе моды преимущественно связаны с одним и тем же каналом рассеяния (некоторая мультипольная гармоника). Используя теорию связанных мод для компактного рассеивателя, рассчитайте сечение рассеяния объекта и продемонстрируйте эффект невидимости. Сохраняется ли эффект при наличии диссипации в объекте?

Примеры экзаменационных билетов в 10-ом семестре:

Билет 1.

1. Построение теории связанных мод для одномодового резонатора с одним портом.
2. Рассмотрите одномодовую резонансную структуру с диссипацией $\gamma\_$ (n-rad), симметрично связанной с двумя каналами рассеяния (моделирующую, например, диэлектрический слой в симметричном окружении). Используя теорию связанных мод для этой структуры, рассчитайте коэффициент поглощения при возбуждении системы из одного канала. Найдите условие максимального поглощения. Каково значение этой максимальной скорости поглощения?

Билет 2.

1. Резонансы диэлектрического слоя.
2. Постройте формальную матрицу рассеяния для слоя вакуума толщиной L в базисе плоских волн распространяющихся по нормали к этому «слою вакуума». Найдите собственные числа данной матрицы рассеяния.