Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Проректор по учебной работе и экономическому развитию
D.A. Zubtsov

Work program of the course (training module)

course: Selected Sections of Higher Mathematics/Избранные главы высшей математики
major: Applied Mathematics and Physics
specialization: Beam-Plasma Systems and Technologies/Пучково-плазменные системы и технологии
Phystech School of Aerospace Technology
Chair of Logistics Systems and Technologies
term: 1
qualification: Master

Semesters, forms of interim assessment:
1 (fall) - Grading test
2 (spring) - Grading test

Academic hours: 120 AH in total, including:
lectures: 0 AH.
seminars: 120 AH.
laboratory practical: 0 AH.

Independent work: 60 AH.

In total: 180 AH, credits in total: 4

Author of the program: S.E. Gorodetskiy, candidate of physics and mathematical sciences

The program was discussed at the Chair of Logistics Systems and Technologies 09.02.2022

Annotation

Discipline belongs to the basic part of the educational program. Mastering the discipline is aimed at developing the ability to acquire new scientific and professional knowledge using modern educational and information technologies. Topics covered include Vector Algebra and elements of Analytic Geometry, Functional Analysis, Elements of Differential Equation and Mathematical Statistics.

1. Study objective

Purpose of the course

1. To provide students with knowledge in the fields of linear algebra, analytic geometry (special attention is given to problems that are to be encountered in further sections of the course).
2. To provide students with knowledge of calculus so that they can explore the behavior of functions of one or several variables, calculate integrals and multiple integrals and know the applications of calculus to field theory.
3. To study specific sections of ordinary differential equations and partial differential equations that are most relevant in beam-plasma systems modeling and analysis.
4. To get acquainted with basics of statistical analysis, regression and correlation analysis and hypothesis testing

Tasks of the course

1. Students deepen their knowledge of analytic geometry and linear algebra.
2. Students obtain skills in calculus that are necessary for applications such as calculating area of a domain in the plane, calculating arc length, calculating surface area, calculating volume of the solid, finding work done on a charged particle moving through electric field, calculating field flow through the surface etc.
3. Students get to know basic types of differential equations used in plasma physics and ways of solving them.
4. Students can use methods of mathematical statistics in experimental data processing.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
Gen.Pro.C-1 Gain fundamental scientific knowledge in the field of physical and mathematical sciences	Gen.Pro.C-1.1 Apply fundamental scientific knowledge in the field of physical and mathematical sciences
	Gen.Pro.C-1.2 Consolidate and critically assess professional experience and research findings
Pro.C-1 Assign, formalize, and solve tasks, develop and research mathematical models of the studied phenomena and processes, systematically analyze scientific problems and obtain new scientific results	Pro.C-1.1 Locate, analyze, and summarize information on current research findings within the subject area
	Pro.C-1.3 Apply theoretical and/or experimental research methods to a specific scientific task and interpret the obtained results

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

Basic types of differential equations used in plasma physics and ways of solving them.

be able to:

To use calculus skills that are necessary for applications such as calculating area of a domain in the plane, calculating arc length, calculating surface area, calculating volume of the solid, finding work done on a charged particle moving through electric field, calculating field flow through the surface etc.

master:

Methods of mathematical statistics in experimental data processing.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Vector algebra		8		4
2	Elements of analytic geometry		8		4
3	Matrices and determinants		8		4
4	Systems of linear equations		8		4
5	Linear operators. Eigenvectors		10		4
6	Quadratic forms		10		6
7	Taylor formula and Taylor series		8		4
8	Graph sketching using derivatives		8		4
9	Maxima and minima of functions of several variables. Conditional extrema		8		4
10	Methods of integration		8		4
11	Multiple integrals		8		4
12	Path integrals. Surface integrals		6		2
13	Elements of ordinary differential equations		6		4
14	Elements of partial differential equations		8		4
15	Elements of mathematical statistics		8		4
AH in total			120		60
Exam preparation		0 AH.			
Total complexity		180 AH., credits in total 4			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 1 (Fall)

1. Vector algebra

Vectors. Dot product. Cross product. Linearly dependent and linearly independent vectors. Basis.

2. Elements of analytic geometry

Equations of lines and planes. Different problems concerning lines and planes (calculating distance between skew lines, volume of tetrahedron, constituting the equation of a plane passing through one line and parallel to the other etc).

3. Matrices and determinants

Operations with matrices. Inverse matrices. Degenerate and non-degenerate matrices. Determinant of a matrix, Its properties and ways of finding it.

4. Systems of linear equations

Homogeneous systems of linear equations. Using elementary row operations for solving systems of linear equations. Non-homogeneous systems of equations. Cramer’s rule.

5. Linear operators. Eigenvectors

Linear operators and their matrices. Transformation matrix. Eigenvectors and eigenvalues. Diagonalization. Jordan form of a matrix.

6. Quadratic forms

Matrix of a quadratic form. Diagonalization. Positive and negative definite quadratic forms. Positive and negative semi-definite quadratic forms. Sylvester's criterion.

7. Taylor formula and Taylor series

Taylor and Maclaurin formulae, Taylor and Maclaurin series. Calculating limits using Taylor expansion and l'Hôpital's rule.

Semester: 2 (Spring)

8. Graph sketching using derivatives

Maxima and minima of functions of one variable. Asymptotes. Convex and concave functions, inflection points. Sketching graphs using the first and second derivatives of a function.

9. Maxima and minima of functions of several variables. Conditional extrema

Functions of several variables. Partial derivatives. Taylor formula. Directional derivative. Gradient. Maxima and minima. Lagrange multipliers.

10. Methods of integration

Antiderivatives. Definite integrals. Integration by parts, integration by substitution. Integrating rational, irrational, trigonometric functions. Trigonometric substitutions.

11. Multiple integrals

Double integrals over rectangles and more general areas. Iterated integrals. Changing variables in double integrals. Polar coordinates. Triple integrals. Spherical and cylindrical coordinates.

12. Path integrals. Surface integrals

Vector fields. Path integrals. Green's formula. Parametric surfaces. Surface area. Surface integrals. Divergence theorem. Stokes' theorem.

13. Elements of ordinary differential equations

Linear ordinary differential equations. Systems of linear differential equations. Homogeneous and non-homogeneous systems of differential equations. Phase plane. Equilibrium. Types of equilibria in two-dimensional systems: node, saddle, focus. Lyapunov stability.

14. Elements of partial differential equations

Wave equation: derivation of the wave equation, general solution and Cauchy's problem's solution. Heat equation, solving heat equation using Fourier series. Diffusion equation. Continuity equation.

15. Elements of mathematical statistics

The normal distribution. The binomial distribution. The Poisson distribution. Confidence intervals. Chi-squared test. Regression and correlation.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

Standard classroom.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Paul Bamberg, Shlomo Sternberg “A Course in Mathematics for Students of Physics”
2. Richard Boddy, Gordon Smith “Statistical Methods in Practice: for Scientists and Technologists”

Additional literature

1. Lynn H. Loomis, Shlomo Sternberg “Advanced Calculus“
2. David Cherney et al “Linear Algebra”

7. List of web resources that are necessary for the course (training module) mastering

Not used

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

Microsoft Office. Internet access.

9. Guidelines for students to master the course

During the semester, the students are to solve a set of topical problems (1-2 per each topic) and present their solutions at the end of semester.

SUPPLEMENT

Assessment funds for course (training module)

major:	Applied Mathematics and Physics
specialization:	Beam-Plasma Systems and Technologies/Пучково-плазменные системы и технологии
	Phystech School of Aerospace Technology
	Chair of Logistics Systems and Technologies
term:	1
qualification:	Master

Semesters, forms of interim assessment:

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Author: S.E. Gorodetskiy, candidate of physics and mathematical sciences

1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
Gen.Pro.C-1 Gain fundamental scientific knowledge in the field of physical and mathematical sciences	Gen.Pro.C-1.1 Apply fundamental scientific knowledge in the field of physical and mathematical sciences
	Gen.Pro.C-1.2 Consolidate and critically assess professional experience and research findings
Pro.C-1 Assign, formalize, and solve tasks, develop and research mathematical models of the studied phenomena and processes, systematically analyze scientific problems and obtain new scientific results	Pro.C-1.1 Locate, analyze, and summarize information on current research findings within the subject area
	Pro.C-1.3 Apply theoretical and/or experimental research methods to a specific scientific task and interpret the obtained results

2. Competency assessment indicators

As a result of studying the course the student should:

know:

Basic types of differential equations used in plasma physics and ways of solving them.

be able to:

To use calculus skills that are necessary for applications such as calculating area of a domain in the plane, calculating arc length, calculating surface area, calculating volume of the solid, finding work done on a charged particle moving through electric field, calculating field flow through the surface etc.

master:

Methods of mathematical statistics in experimental data processing.

3. List of typical control tasks used to evaluate knowledge and skills

The current control is applied in the following forms:

- evaluation of answers to questions in the process of a short (up to 5 minutes) selective oral survey before the start of each lesson based on the materials of the previous lesson;
- assessment of the ability to solve at the blackboard and/or in writing typical examples and/or tasks considered in classes;
- assessment of activity and answers to questions when solving typical tasks in accordance with the program of practical classes.

4. Evaluation criteria

1. Vectors. Linearly dependent and linearly independent vectors. Basis. Dimension. Scalar product and its properties. Orthogonal projections of vectors.
2. Vector production and its properties. The mixed product and its properties. The formula of the double vector product.
3. A straight line on a plane and in space. Various ways to set a straight line.
4. A plane in space. Different kinds of plane equation.
5. Metric problems about straight lines and planes in space.
6. Multiplication and inversion of matrices. Elementary transformations of matrices. The rank of the matrix.
7. Definition and basic properties of matrix determinants. The formula of decomposition by row (column). The formula of complete decomposition.
8. Systems of linear equations. The Gauss method. The criterion of compatibility. Type of general solution.
9. Linear space. Subspaces. Linear mappings (operators).
10. The problem of eigenvectors and eigenvalues of linear mapping.
11. Bilinear functions and quadratic forms. Reduction of a quadratic form to a canonical form. The familiarity of the quadratic form.

12. Higher order derivatives. The Leibniz formula.
13. The Lopital rule of uncertainty disclosure.
14. The Taylor formula.
15. The study of functions using a derivative. Plotting graphs.
16. Extremes of functions of many variables. Necessary conditions. Sufficient conditions.
17. Conditional extremum.
18. Indefinite integral. A definite Riemann integral. Integration of some functions.
19. Multiple Riemann integral. Reduction of double and triple integrals to repeated ones.
20. Replacement of variables in a multiple integral.
21. Curvilinear integral of the first and second kind.
22. The surface integral of the first and second kind.
23. Green's formula on the plane.
24. The Stokes formula.
25. The Gauss–Ostrogradsky theorem.
26. Elements of field theory. Gradient. Divergence. The rotor. The operator “nabla" and actions with it.
27. Linear differential equations of the first order.
28. Linear differential equations with constant coefficients.
29. Systems of linear differential equations with constant coefficients.
30. Autonomous systems of linear differential equations. Equilibrium positions.
31. Stability and asymptotic stability of equilibrium positions.
32. The nature of the behavior of phase trajectories in the vicinity of equilibrium positions of two-dimensional autonomous systems of differential equations.

The mark is excellent 10 points - given to a student who has shown comprehensive, systematized, deep knowledge of the curriculum of the discipline, who is interested in this subject area, who has demonstrated the ability to confidently and creatively apply them in practice in solving specific problems, free and correct justification of the decisions made.

An excellent mark of 9 points is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to confidently apply them in practice in solving specific problems, free and correct justification of decisions made.

An excellent grade of 8 points is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to confidently apply them in practice in solving specific problems, the correct justification of the decisions made, with some drawbacks.

A good score of 7 points is given to a student if he firmly knows the material, expresses it competently and to the point, knows how to apply the knowledge gained in practice, but does not adequately substantiate the results obtained.

A good score of 6 points is given to a student if he firmly knows the material, expounds it competently and to the point, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems.

A good score of 5 points is given to a student if he basically knows the material, expresses it competently and to the point, knows how to apply the knowledge gained in practice, but makes a large number of inaccuracies in the answer or in solving problems.

The mark is satisfactory 4 points - given to a student who has shown a fragmentary, scattered nature of knowledge, insufficiently correct formulations of basic concepts, a violation of the logical sequence in the presentation of the program material, but at the same time he has mastered the main sections of the curriculum necessary for further education and can apply the acquired knowledge in sample in a standard situation.

The mark is satisfactory 3 points - given to a student who has shown a fragmented, scattered nature of knowledge, makes mistakes in the formulation of basic concepts, disruptions in the logical sequence in the presentation of program material, poorly knows the main sections of the curriculum necessary for further education and hardly applies the acquired knowledge even in standard situations.

The score is unsatisfactory 2 points - given to a student who does not know most of the main content of the curriculum of the discipline, makes gross errors in the formulation of basic principles and does not know how to use the knowledge gained when solving typical problems.

The mark is unsatisfactory 1 point - given to a student who does not know the main content of the curriculum of the discipline, makes gross errors in the formulation of the basic concepts of the discipline and generally does not have the skills to solve typical practical problems.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

Differentiated credit is carried out orally according to tickets approved by the head of the department. The examiner is given the right, in addition to the theoretical questions of the ticket, to give students tasks and examples, typical versions of which were considered in practical classes. Students with the permission of the examiner can use notes, seminars, reference literature only during preparation. Time is allocated for preparation and survey in accordance with the approved standards.