Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Head of the Phystech School of Applied Mathematics and Informatics
A.M. Raygorodskiy

Work program of the course (training module)

course:	Game Theory/Теория игр
major:	Applied Mathematics and Physics
specialization:	Applied Bioinformatics/Прикладная биоинформатика
	Phystech School of Biological and Medical Physics
	Chair of Discrete Mathematics
term:	1
qualification:	Master

Semester, form of interim assessment: 1 (fall) - Exam

Academic hours: 75 AH in total, including:
lectures: 30 AH.
seminars: 45 AH.
laboratory practical: 0 AH.

Independent work: 75 AH.

Exam preparation: 30 AH.

In total: 180 AH, credits in total: 4

Author of the program: D.V. Musatov, candidate of physics and mathematical sciences, associate professor

The program was discussed at the Chair of Discrete Mathematics 05.03.2020

Annotation

The course is intended for second-year master students. It introduces main concepts of game theory, especially about static, dynamic and cooperative games and various equilibrium notions in these frameworks. Several special topics are also included.

1. Study objective

Purpose of the course

To acquaint students with the basic concepts and results of non-cooperative and cooperative game theory. The central place in the course is occupied by the concept of Nash equilibrium, sequential equilibrium, and also the concept of a kernel in cooperative games with side payments.

Tasks of the course

- Students mastering basic knowledge (concepts, concepts, methods and models) in game theory;
- acquisition of theoretical knowledge and practical skills in game theory;
- providing advice and assistance to students in conducting their own theoretical research in game theory.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-1 Use a systematic approach to critically analyze a problem,and develop an action plan	UC-1.3 Develop a step-by-step strategy for achieving a goal, foresee the result of each step, evaluate the overall impact on the planned activity and its participants
Gen.Pro.C-1 Gain fundamental scientific knowledge in the field of physical and mathematical sciences	Gen.Pro.C-1.1 Apply fundamental scientific knowledge in the field of physical and mathematical sciences
Gen.Pro.C-2 Acquire an understanding of current scientific and technological challenges in professional settings, and scientifically formulate professional objectives	Gen.Pro.C-2.1 Assess the current state of mathematical research within professional settings
Pro.C-1 Assign, formalize, and solve tasks, develop and research mathematical models of the studied phenomena and processes, systematically analyze scientific problems and obtain new scientific results	Pro.C-1.3 Apply theoretical and/or experimental research methods to a specific scientific task and interpret the obtained results

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- Fundamental concepts, laws, game theory;
- modern problems of the relevant sections of game theory;
- concepts, axioms, methods of proof and proof of the main theorems in the sections included in the basic part of the game theory cycle;
- basic properties of the corresponding mathematical objects;
- analytical and numerical approaches and methods for solving typical applied problems of game theory.

be able to:

- Understand the task;
- use your knowledge to solve fundamental and applied problems;
- evaluate the correctness of the problem statements;
- strictly prove or disprove the statement;
- independently find algorithms for solving problems, including non-standard ones, and conduct their analysis;
- independently see the consequences of the results;
- accurately represent mathematical knowledge in topology orally and in writing.

master:

- Skills of mastering a large amount of information and solving problems (including complex ones);
- skills of independent work and mastering new disciplines;
- the culture of the formulation, analysis and solution of mathematical and applied problems that require the use of mathematical approaches and methods for their solution;
- the subject language of topology and the skills of competent description of problem solving and presentation of the results.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	What is a game? Games in normal form. Matrix and bimatrix games.	6	9		15
2	Mixed Nash equilibria.	6	9		15
3	Dynamic games with complete information, asymmetric case	6	9		15
4	Repeated games.	6	9		15
5	Basics of social choice theory.	6	9		15
AH in total		30	45		75
Exam preparation		30 AH.			
Total complexity		180 AH., credits in total 4			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 1 (Fall)

1. What is a game? Games in normal form. Matrix and bimatrix games.

Dominance in static games.
Pure Nash equilibria.

2. Mixed Nash equilibria.

Algorithms for finding Nash equilibria.
Dynamic games with complete information, symmetric case

3. Dynamic games with complete information, asymmetric case

Dynamic games with incomplete information Signaling games.

4. Repeated games.

Cooperative games. Stable matchings.

5. Basics of social choice theory.

Fair division. Basics of auction theory.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

A standard classroom.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Лекции по теории игр и экономическому моделированию [Текст] : [учеб. пособие для вузов] / И. С. Меньшиков .— 2-е изд., испр. и доп. — М. : Контакт Плюс, 2010 .— 336 с.

2. Теория графов в управлении организационными системами [Текст] / В. Н. Бурков, А. Ю. Заложнев, Д. А. Новиков ; Рос. акад. наук, Ин-т проблем управления им. В. А. Трапезникова - М.СИНТЕГ,2001

Additional literature

1. Игры для развития системного мышления [Текст] / Л. Бут Свини, Д. Медоуз; пер. с англ. Е. С. Оганесян; под ред. Н. П. Тарасовойd - М.БИНОМ. Лаборатория знаний,2014

7. List of web resources that are necessary for the course (training module) mastering

http://dm.fizteh.ru/

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

Multimedia technologies can be employed during lectures and practical lessons, including presentations.

9. Guidelines for students to master the course

1. It is recommended to successfully pass the test papers, as this simplifies the final certification in the subject.
2. To prepare for the final certification in the subject, it is best to use the lecture materials.

SUPPLEMENT

Assessment funds for course (training module)

major:	Applied Mathematics and Physics
specialization:	Applied Bioinformatics/Прикладная биоинформатика Phystech School of Biological and Medical Physics Chair of Discrete Mathematics
term:	1
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Semester, form of interim assessment: 1 (fall) - Exam

Author: D.V. Musatov, candidate of physics and mathematical sciences, associate professor

1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
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Gen.Pro.C-2 Acquire an understanding of current scientific and technological challenges in professional settings, and scientifically formulate professional objectives	Gen.Pro.C-2.1 Assess the current state of mathematical research within professional settings
Pro.C-1 Assign, formalize, and solve tasks, develop and research mathematical models of the studied phenomena and processes, systematically analyze scientific problems and obtain new scientific results	Pro.C-1.3 Apply theoretical and/or experimental research methods to a specific scientific task and interpret the obtained results

2. Competency assessment indicators

As a result of studying the course the student should:

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- Fundamental concepts, laws, game theory;
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- Understand the task;
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3. List of typical control tasks used to evaluate knowledge and skills

Example home assignment problems

1. A matrix of a static game is given. Sequentially exclude dominated strategies and then solve the remaining game 2×N.

2. Find all equilibria in a stone-scissors-paper-like game with given number of figures and gicen rules of figures beating each other.

3. 50 pirates ordered by rank are dividing a big treasure by the following procedure. The captain offers a division. If at least half of the crew (including the captain) agree then the division is applied. Otherwise the captain is thrown overboard and the second-ranked pirate offers a division, and so on. Formalize the situation as a dynamic game and find a subgame perfect Nash equilibrium.

4. Find the core and the Shapley value in some cooperative game.

5. Find all stable matchings at some matching market.

4. Evaluation criteria

Questions for the exam:

1. The definition of the game in normal form: strategy, player, utility. Nash equilibrium in pure strategies. Examples. Prisoner's dilemma. The game "rock-paper-scissors."

2. Definition of a mixed strategy. Nash equilibrium in mixed strategies.

3. Brauer theorem. Sperner's lemma. Kakutani theorem. Nash's theorem on the existence of equilibrium in mixed strategies.

4. Dominant strategies. The consistent exclusion of highly dominated strategies. Minimax and Maximin. Zero-sum games. Saddle point.

5. The expanded form of the game. Equivalence with normal form. Equilibria committed in sub-games. Examples.

6. Weak and strong sequential Nash equilibria. The faith of the players.

7. Introduction to the theory of cooperative games. The concept of equilibrium. The concept of the kernel. Vector Shapley.

8. Dynamic games. Finite and endless repeating games. The prisoner's endless dilemma. The principle of a single deviation. Popular theorem. Duels. Simultaneous Truels.

Exam ticket examples:

Ticket 1.

1. Brauer theorem. Sperner's lemma. Kakutani theorem.

2. The concept of equilibrium. The concept of the kernel. Vector Shapley.

Ticket 2.

1. Definition of a mixed strategy. Nash equilibrium in mixed strategies.

2. Dynamic games. Finite and endless repeating games.

Assessment “excellent (10)” is given to a student who has displayed comprehensive, systematic and deep knowledge of the educational program material, has independently performed all the tasks stipulated by the program, has deeply studied the basic and additional literature recommended by the program, has been actively working in the classroom, and understands the basic scientific concepts on studied discipline, who showed creativity and scientific approach in understanding and presenting educational program material, whose answer is characterized by using rich and adequate terms, and by the consistent and logical presentation of the material;

Assessment “excellent (9)” is given to a student who has displayed comprehensive, systematic knowledge of the educational program material, has independently performed all the tasks provided by the program, has deeply mastered the basic literature and is familiar with the additional literature recommended by the program, has been actively working in the classroom, has shown the systematic nature of knowledge on discipline sufficient for further study, as well as the ability to amplify it on one’s own, whose answer is distinguished by the accuracy of the terms used, and the presentation of the material in it is consistent and logical;

Assessment “excellent (8)” is given to a student who has displayed complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently performed all the tasks stipulated by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (7)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in the answer, has independently performed all the tasks provided by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (6)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently carried out the main tasks stipulated by the program, studied the basic literature recommended by the program, showed systematic character of his knowledge of the discipline, which is sufficient for further study;

Assessment “good (5)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, mastered the basic literature recommended by the program, made some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors by himself;

Assessment “satisfactory (4)” is given to a student who has discovered knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, learned the main literature but allowed some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors under the guidance of a teacher;

Assessment “satisfactory (3)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, not showed activity in the classroom, independently fulfilled the main tasks envisaged by the program, but allowed errors in their implementation and in the answer during the test, but possessing necessary knowledge for elimination under the guidance of the teacher of the most essential errors;

Assessment “unsatisfactory (2)” is given to a student who showed gaps in knowledge or lack of knowledge on a significant part of the basic educational program material, who has not performed independently the main tasks demanded by the program, made fundamental errors in the fulfillment of the tasks stipulated by the program, who is not able to continue his studies or start professional activities without additional training in the discipline in question;

Assessment “unsatisfactory (1)” is given to a student when there is no answer (refusal to answer), or when the submitted answer does not correspond at all to the essence of the questions contained in the task.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

During examination the student are allowed to use the program of the discipline.