Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Vice Rector for Academic Affairs

A.A. Voronov

Work program of the course (training module)

course: Introduction to Mathematical Analysis/Введение в математический анализ
major: Information Science and Computer Engineering
specialization: Computer Science/Информатика
Phystech School of Applied Mathematics and Informatics
Chair of Higher Mathematics
term: 1
qualification: Bachelor

Semester, form of interim assessment: 1 (fall) - Exam

Academic hours: 90 AH in total, including:
lectures: 30 AH.
seminars: 60 AH.
laboratory practical: 0 AH.

Independent work: 105 AH.

Exam preparation: 30 AH.

In total: 225 AH, credits in total: 5

Number of course papers, tasks: 5

Authors of the program:

A.I. Dnestryan, candidate of physics and mathematical sciences, associate professor, associate professor
R.D. Zukhba, старший преподаватель
O.E. Orel, candidate of physics and mathematical sciences, associate professor, associate professor

The program was discussed at the Chair of Higher Mathematics 20.05.2021

Annotation

Discipline belongs to the basic part of the educational program. Mastering the discipline is aimed at developing the ability to acquire new scientific and professional knowledge using modern educational and information technologies. Topics covered include Algebraic Equations and Inequalities, Progressions, Trigonometry, Exponential and Logarithmic Functions, Complex Numbers, Real Numbers, Sequence Limits, Limit and Continuity of Functions of One Variable.

1. Study objective

Purpose of the course

Formation of basic knowledge in mathematical analysis for further use in other areas of mathematical knowledge and disciplines with natural science content; the formation of a mathematical culture, research skills and the ability to apply knowledge in practice.

Tasks of the course

- Acquisition of theoretical knowledge and practical skills by students in the field of the theory of limits, differential and integral calculus, the theory of series;
- preparing students for the study of related mathematical disciplines;
- acquisition of skills in the application of methods of mathematical analysis in physics and other natural science disciplines

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically assess and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan actions required to solve them
	UC-1.2 Find, critically assess, and select information required for the task in hand
	UC-1.3 Consider various options for solving a problem, assess the advantages and disadvantages of each option
	UC-1.4 Make competent judgments and estimates supported by logic and reasoning
UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- basic properties of the limits of sequences and functions of a real variable, derivative, differential, indefinite integral; properties of functions that are continuous on a segment;
- basic "remarkable limits", tabular formulas for derivatives, formulas for differentiation.

be able to:

- write down statements using logical symbols;
- calculate the limits of sequences and functions of a real variable;
- calculate the derivatives of elementary functions, expand elementary functions according to the Taylor formula; calculate the limits of functions using L'Hôpital's rule.

master:

- the subject language of classical mathematical analysis, used in the construction of the theory of limits;
- the apparatus of the theory of limits, differential and integral calculus for solving various problems arising in physics, technology, economics and other applied disciplines.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Algebraic equations and inequalities	1	2		10
2	Progression	1	2		10
3	Trigonometry	4	8		15
4	Exponential and logarithmic functions	4	4		10
5	Method of mathematical induction	1	2		8
6	Real numbers	4	10		15
7	Sequence limits	8	16		20
8	Limit and continuity of functions	7	16		17
AH in total		30	60		105
Exam preparation		30 AH.			
Total complexity		225 AH., credits in total 5			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 1 (Fall)

1. Algebraic equations and inequalities

Quadratic equations. Vieta's theorem. Graph of a quadratic function. Biquadratic equations. The main theorem of algebra. Method of intervals for solving inequalities

2. Progression

Arithmetic and geometric progressions. Sum of arithmetic and geometric progression.

3. Trigonometry

Unit circle, trigonometric functions of an arbitrary argument. Trigonometric formulas. Trigonometric function graphs. The simplest trigonometric equations and inequalities

4. Exponential and logarithmic functions

Exponential function, its properties and graph. Logarithms. Number e and natural logarithm. Exponential equations and inequalities. Logarithmic function, logarithmic equations and inequalities.

5. Method of mathematical induction

Formulation of the principle of mathematical induction. Proofs of equalities, inequalities and various statements from elementary algebra and geometry.

6. Real numbers

Real numbers. Inequality relations between real numbers. Archimedes property. The density of the set of real numbers. The theorem on the existence and uniqueness of the exact upper (lower) bound on a numerical set bounded above (below). Arithmetic operations with real numbers. Infinite decimal representation of real numbers. The countability of the set of rational numbers, the uncountability of the set of real numbers.

7. Sequence limits

Limit of a numerical sequence. Cantor's nested line segment theorem. Uniqueness of the limit. Infinitesimal sequences and their properties. Limit properties related to inequalities. Arithmetic operations with converging sequences. Weierstrass' theorem on the limit of a monotone bounded sequence. The number e. Infinitely large sequences and their properties.
Subsequences, partial limits. Upper and lower limits of a number sequence. Bolzano-Weierstrass theorem. Cauchy's criterion for the convergence of a sequence.

8. Limit and continuity of functions

Theorem on the one-sided limit of a monotone function. Weierstrass' extreme value theorem. Intermediate value theorem. Uniform continuity.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

Classroom equipped with a multimedia projector, screen and microphone.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Advanced calculus, A. Friedman ; The Ohio State University. Mineola ; New York, Dover publications, inc., 2016
2. Mathematical analysis I /V. A. Zorich. Berlin, Springer, 2015

Additional literature

1. Лекции по математическому анализу [Текст] : [в 2 ч.] : учеб. пособие для вузов. Ч. 1 / Г. Н. Яковлев .— 2-е изд., перераб. и доп. — М. : Физматлит, 2004 .— 340 с.
2. Краткий курс математического анализа [Текст] : в 2 т. Т. 2 : Дифференциальное и интегральное исчисления функций многих переменных. Гармонический анализ : учеб. пособие для вузов / Л. Д. Кудрявцев .— 3-е изд., перераб. — М. : Физматлит, 2008 .— 424 с.

7. List of web resources that are necessary for the course (training module) mastering

http://www.math.mipt.ru
https://library.harvard.edu/

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

The lectures use multimedia technologies, including the demonstration of presentations.

9. Guidelines for students to master the course

Provided in the annually developed homework assignments.

SUPPLEMENT

Assessment funds for course (training module)

major:	Information Science and Computer Engineering
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	Phystech School of Applied Mathematics and Informatics
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1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically assess and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan actions required to solve them
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	UC-1.4 Make competent judgments and estimates supported by logic and reasoning
UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

2. Competency assessment indicators

As a result of studying the course the student should:

know:

- basic properties of the limits of sequences and functions of a real variable, derivative, differential, indefinite integral; properties of functions that are continuous on a segment;
- basic "remarkable limits", tabular formulas for derivatives, formulas for differentiation.

be able to:

- write down statements using logical symbols;
- calculate the limits of sequences and functions of a real variable;
- calculate the derivatives of elementary functions, expand elementary functions according to the Taylor formula; calculate the limits of functions using L'Hôpital's rule.

master:

- the subject language of classical mathematical analysis, used in the construction of the theory of limits;
- the apparatus of the theory of limits, differential and integral calculus for solving various problems arising in physics, technology, economics and other applied disciplines.

3. List of typical control tasks used to evaluate knowledge and skills

Current control based on homework is carried out during the academic semester within the timeframe established by the Educational Department, in accordance with the curriculum.

To pass the assignment, the student must provide a solution to the homework problem in writing, answer the teacher's questions and write a test on the assignment, which tests the knowledge of concepts and statements on the topics of the assignment being handed over and the ability to solve problems.

During the execution of the test, you cannot use the help of other persons, computers and mobile phones.

4. Evaluation criteria

Certification in the discipline "Introduction of Mathematical Analysis/Введение в математический анализ" is carried out in the form of an exam. The exam is conducted in accordance with the control tasks previously completed by the students.

Control tasks:

1. Solve the equation x2 + 5x + 6 = 0 using Vieta's theorem.
2. Does the graph of the function f (x) = x2 + x + 1 intersect the abscissa?

3. How many solutions can the equation x3 + ax2 + bx + c = 0 have?
4. It is known that for any n the sum of the first n terms of the arithmetic progression is 4n2 - 3n. Find the tenth term of this progression.
5. Draw (approximately) an angle of 1000 °.
6. Find the 300 ° radial angle measure.
7. Find sinx if you know that the angle x lies in the first quarter and cos x = 0.6.
8. Calculate the sin 15 °.
9. Solve the equation sin 2x = 3 sinx.
10. Solve the equation logx (x + 2) = 2.
11. Solve the equation z3 = 1 over the field of complex numbers.
12. Let X, Y be non-empty numerical sets, and inf X> sup Y. Can X and Y intersect?
13. Formulate a positive form: a) the sequence is not limited; b) the number a is not the limit of the sequence.
14. Prove that the convergence of the sequence {an} implies the convergence of the sequence {| an |}. Is the opposite true?
15. Let {an} be a converging sequence. Is the sequence {an + 1 - an} converging?
16. Let the sequence {an} converge, and the sequence {bn} diverge. What can be said about the convergence of the sequences {an + bn}, {anbn}?
17. Can we say that: a) each infinitely large sequence is unbounded; b) is every unbounded sequence infinitely large?
18. Prove that if some subsequence of a monotone sequence is bounded, then the sequence itself is bounded as well.
19. Build an example of a sequence that a) has no finite partial limits; b) has a single finite partial limit, but does not converge.
20. Can the set of partial limits of a sequence be innumerable? Same as (0,1)?

Examples of exam tickets:

Ticket 1
1. Solve the equation x2 + 5x + 6 = 0 using Vieta's theorem.
2. Can it be argued that:
a) each infinitely large sequence is unbounded;
b) any unbounded sequence is infinitely large?

Ticket 2
1. Does the graph of the function f (x) = x2 + x + 1 intersect the abscissa?
2. Build an example sequence that:
a) has no finite partial limits;
b) has a single finite partial limit, but does not converge.

Grade "excellent (10)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks;

Grade "excellent (9)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently found and corrected;

Grade "excellent (8)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently corrected after the instructions of an examiner;

Grade "good (7)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made minor mistakes when answering questions or solving problems;

Grade "good (6)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made rare mistakes when answering questions or solving problems;

Grade "good (5)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made mistakes when answering questions or solving problems;

Grade "satisfactory (4)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, but understands the subject well, is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "satisfactory (3)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, has inconsistencies in understanding the course, but is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "unsatisfactory (2)" is given to a student who does not possess knowledge of the essential concept of the course, has made gross mistakes in formulations of basic concepts and cannot use the knowledge in solving typical tasks;

Grade "unsatisfactory (1)" is given to a student who has exhibited total lack of knowledge of the course.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

When conducting an oral exam, the student is given 1 astronomical hour for preparation. The poll of a student on a ticket for an oral exam should not exceed 2 astronomical hours.

During the exam, students can use only the discipline program.