Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Head of the Phystech School of Applied Mathematics and Informatics
A.M. Raygorodskiy

Work program of the course (training module)

course:	Fundamentals of Topology/Основы топологии
major:	Information Science and Computer Engineering
specialization:	Computer Science/Информатика
	Phystech School of Applied Mathematics and Informatics
	Chair of Discrete Mathematics
term:	4
qualification:	Bachelor

Semester, form of interim assessment: 7 (fall) - Grading test

Academic hours: 60 AH in total, including:
lectures: 30 AH.
seminars: 30 AH.
laboratory practical: 0 AH.

Independent work: 75 AH.

In total: 135 AH, credits in total: 3

Number of course papers, tasks: 2

Author of the program: G.G. Gusev, candidate of physics and mathematical sciences, associate professor, associate professor

The program was discussed at the Chair of Discrete Mathematics 04.06.2020

Annotation

The discipline acquaints students with the basic concepts and constructions of topology, teaches them to apply them in practice to solve specific problems.

1. Study objective

Purpose of the course

- mastering the main modern methods of topology.

Tasks of the course

- Mastering by students of basic knowledge (concepts, concepts, methods and models) in topology;
- acquisition of theoretical knowledge and practical skills in topology;
- providing advice and assistance to students in conducting their own theoretical research in topology.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically assess and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan actions required to solve them
	UC-1.2 Find, critically assess, and select information required for the task in hand
	UC-1.3 Consider various options for solving a problem, assess the advantages and disadvantages of each option
	UC-1.5 Identify and evaluate practical consequences of possible solutions to a problem
	UC-1.4 Make competent judgments and estimates supported by logic and reasoning
Gen.Pro.C-1 Apply fundamental knowledge acquired in the physical and mathematical fields and/or natural sciences and use it in professional settings	Gen.Pro.C-1.1 Analyze the task in hand, outline the ways to complete it
	Gen.Pro.C-1.2 Build mathematical models, make quantitative measurements and estimates
	Gen.Pro.C-1.3 Determine the applicability limits of the obtained results
Gen.Pro.C-4 Collect and process scientific and technical and/or technological data for fundamental and applied problem-solving	Gen.Pro.C-4.1 Apply scientific research and intellectual analysis methods for professional problem-solving
	Gen.Pro.C-4.2 Search for primary sources of scientific and technical and/or technological information in professional settings
	Gen.Pro.C-4.3 Prepare abstracts, reports, bibliographies, and reviews of information in professional settings
	Gen.Pro.C-4.4 Use computer and network skills to obtain, store, and process scientific (technical, technological) information
Pro.C-1 Assign, formalize, and solve tasks, develop and research mathematical models of the studied phenomena and processes, systematically analyze scientific problems, obtain new scientific outcomes	Pro.C-1.3 Apply theoretical and/or experimental research methods to a specific scientific task and interpret the obtained results
	Pro.C-1.2 Make hypotheses, build mathematical models of the studied phenomena and processes, evaluate the quality of the developed model
	Pro.C-1.1 Locate, analyze, and summarize information on current research findings within the subject area

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- Fundamental concepts, laws, theory of topology;
- modern problems of the relevant sections of topology;
- concepts, axioms, methods of proofs and proofs of theorems in the sections included in the basic part of the topology cycle;
- basic properties of the corresponding mathematical objects;
- analytical and numerical approaches and methods for typical applied topology problems.

be able to:

- Understand the task at hand;
- use your knowledge to solve fundamental and applied problems;
- evaluate the correctness of the problem setting;
- strictly prove or disprove the statement;
- independently find algorithms for solving problems, including non-standard ones, and analyze them;
- independently see the consequences of the results obtained;
- accurately represent mathematical knowledge in topology orally and in writing.

master:

- Skills of mastering a large amount of information and solving topology problems (including complex ones);
- skills of independent work and mastering new disciplines;
- culture of formulation, analysis and solution of mathematical and applied problems that require the use of mathematical approaches and methods for their solution;
- the subject language of topology and the skills of competently describing the solution of problems and presenting the results obtained.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Topological spaces.	6	6		15
2	Linear connectivity.	6	6		16
3	Hausdorff property.	6	6		14
4	One-dimensional and two-dimensional manifolds.	6	6		15
5	Planarity of graphs and flat graphs.	6	6		15
AH in total		30	30		75
Exam preparation		0 AH.			
Total complexity		135 AH., credits in total 3			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 7 (Fall)

1. Topological spaces.

Open and closed sets. Continuous displays.

2. Linear connectivity.

Fundamental group of space with selected point.

3. Hausdorff property.

The concept of a cell complex. Various options for the formulation of axiom (W).

4. One-dimensional and two-dimensional manifolds.

Classification of one-dimensional manifolds.

5. Planarity of graphs and flat graphs.

Proof of nonplanarity of graphs K5 and K3,3.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

Standard classroom.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Алгебраическая топология с геометрической точки зрения [Текст] / А. Б. Скопенков - М.МЦНМО,2015
2. Основы комбинаторной топологии [Текст]/Л. С. Понтрягин, -М., Едиториал УРСС, 2004
1. А.С. Мищенко, А.Т. Фоменко. Курс дифференциальной геометрии и топологии. - М., Факториал пресс, 2000 - 448 с.

Additional literature

1. Сборник задач по дифференциальной геометрии и топологии [Текст] : [учеб. пособие для вузов] / А. С. Мищенко, Ю. П. Соловьев, А. Т. Фоменко ; под общ. ред. А. Т. Фоменко .— М : Физматлит, 2004 .— 412 с.

7. List of web resources that are necessary for the course (training module) mastering

http://dm.fizteh.ru/
http://web.stanford.edu/class/ee364b/lectures.html

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

The current control consists of two tests per semester, as well as oral delivery of assignments for independent decision. Evaluation criteria are attached. Also attached is an example of a test task and several problems for independent solution on various topics at the end of the program.

9. Guidelines for students to master the course

1. It is recommended to successfully pass test papers, as this simplifies the final certification in the subject.
2. To prepare for the final certification in the subject, it is best to use the lecture materials.

SUPPLEMENT

Assessment funds for course (training module)

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Semester, form of interim assessment: 7 (fall) - Grading test

Author: G.G. Gusev, candidate of physics and mathematical sciences, associate professor, associate professor

1. Competencies formed during the process of studying the course

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2. Competency assessment indicators

As a result of studying the course the student should:

know:

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- culture of formulation, analysis and solution of mathematical and applied problems that require the use of mathematical approaches and methods for their solution;
- the subject language of topology and the skills of competently describing the solution of problems and presenting the results obtained.

3. List of typical control tasks used to evaluate knowledge and skills

The current control consists of two tests per semester, as well as oral delivery of assignments for independent decision. Evaluation criteria are attached. Also attached is an example of a test task and several problems for independent solution on various topics at the end of the program.

4. Evaluation criteria

Questions for credit:

1. Topological spaces, open and closed sets, continuous maps.
2. Linear connectivity. Path homotopy - definition, symmetry and transitivity.
3. Product of paths - independence from the choice of the equivalence class. Determination of the fundamental group of a space with a distinguished point.
4. Associativity of the fundamental group.
5. The existence of a unit and an inverse element in the fundamental group.

Calculation of the fundamental group of a convex set.

Circle fundamental group with proof.

6. Applications of the fundamental group: Brouwer's theorem, The Main Theorem of Algebra.
7. Hausdorff property of a topological space, an example of a non-Hausdorff space. The concept of a cell complex. The image under the characteristic map coincides with the closure of the cell, even if the axiom (W) is not fulfilled - proof.
8. Various versions of the formulation of axiom (W), their equivalence (with a complete proof). An example of a complex that does not satisfy axiom (W).
9. Cellular partitions of one-dimensional complexes, spheres and disks.
10. Cellular partitions of two-dimensional surfaces.
11. The concept of a subcomplex, a d-dimensional skeleton, a locally finite complex. Partitioning into cells of the Cartesian product of two cell complexes. A sufficient condition for the resulting partition to be a cellular complex (no proof).
12. A necessary and sufficient condition for the continuity of a mapping from a cell complex to a topological space (with proof).
13. Borsuk's theorem (ideas of proof). If the subcomplex is contractible, then the quotient of the complex with respect to it is homotopy equivalent to the original complex (with a proof).
14. Isomorphism of fundamental groups of a cellular space and its two-dimensional skeleton. Calculation of the fundamental group of a cellular space X with a single vertex (formulation, no proof).
15. Cell homology definition of a chain complex and its homology.
16. Cellular homology of one-dimensional complexes, spheres and disks - calculation through cell division.
17. Cellular homology of two-dimensional surfaces - calculation through cell decomposition.
18. Definition of a 71-dimensional topological manifold without boundary. Classification of 0-dimensional and 1-dimensional connected manifolds.
19. Homogeneous 2-dimensional simplicial complex as a topological space glued from triangles along the sides. Triangulation theorem for a 2-manifold (without proof).
20. The dual graph of a triangulated 2-manifold. Cutting the triangulation using the spanning tree of the dual graph. Lemma: any 2-manifold is homeomorphic to a 2n-gon, the sides of which are split into pairs, and each pair of sides is glued together in one of two possible ways.
21. Lemmas about gluing polygons: pasting a Mobius strip, handles. Proof of the theorem on the classification of connected compact two-dimensional manifolds.

Topics for term papers:

1. Axiomatic definition of the additive Euler characteristic. Calculation of the Euler characteristics of an arbitrary union of cells of a finite cell complex.
2. Planarity of graphs and flat graphs. The concept of a face of a plane graph, Euler's formula (with proof). Topological interpretation of the formula (Euler characteristic).

- the mark "excellent (10)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to confidently apply them in practice in solving specific problems, free and correct justification of the decisions made

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-the mark "excellent (8)" is given to a student who has shown a comprehensive, systematized, deep knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, and the correct justification of the decisions made

- the mark "good (7)" is given to a student if he knows the material well, expresses it competently and in essence, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "good (6)" is given to a student if he knows the material, presents it competently and in essence, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "good (5)" is given to the student if he knows the material, and essentially expounds it, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "satisfactory (4)" is given to a student who has shown a fragmented, scattered nature of knowledge, insufficiently correct formulations of basic concepts, a violation of the logical sequence in the presentation of the program material, but at the same time he owns the main sections of the curriculum necessary for further education and can apply the received knowledge of the pattern in a standard situation;

- the mark "satisfactory (3)" is given to a student who has shown a fragmented, scattered nature of knowledge, insufficiently correct formulations of basic concepts, a violation of the logical sequence in the presentation of program material, but at the same time he has fragmentary knowledge of the main sections of the curriculum necessary for further education and can apply the knowledge gained by the model in a standard situation;

- the mark "unsatisfactory (2)" is given to a student who does not know most of the main content of the curriculum of the discipline, makes gross errors in the formulation of the basic concepts of the discipline and does not know how to use the knowledge gained in solving typical practical problems

- the mark "unsatisfactory (1)" is given to a student who does not know the wording of the basic concepts of the discipline

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

During the test, students can use the discipline program.